

Computing Exact Power for Multivariate Repeated Measurements Design

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ABSTRACT

Repeated measurements often arise in medical and laboratory research when investigators are constrained by costs or time to give all study treatments to each experimental unit/participant one at a time. In this paper, we derive a generalized method for computing the exact power for a repeated measurements design given the simple case of no learning effect.

INTRODUCTION

Given the case of (t) treatments applied in sequence to (n) experimental units following a washout period, the Hotelling's T^2 -statistic may be used to test the equality of mean treatment effects. If the null hypothesis is rejected, then simultaneous confidence intervals on all contrast derivable from the (t) treatments may be constructed to determine which treatments differ.

METHODOLOGY

Let \bar{y} denote the vector of treatment means, $S_{t \times t}$ the sample covariance matrix and $C_{(t-1) \times t}$ the matrix of invariant contrasts. A direct extension of the univariate to multivariate space (Kelsey, et al., 1996; Kramer, 1972) gives the exact power for the simple repeated measurements design as

$$T_{\text{power}} = \sqrt{(n(\bar{c}\bar{y}')(\mathbf{cSc}')^{-1}(\bar{c}\bar{y}')) - \frac{(t-1)(n-1)}{(n-t+1)} F_{t-1, n-t+1}}, \quad (1)$$

where $F_{(t-1, n-t+1)}$ is the $100(1-\alpha)$ centile of the F-distribution with (t-1) and (n-t+1) degrees of freedom (Johnson and Wichern, 1982). When t=2, we see that equation number (1) gives the exact power for the univariate case.

EXAMPLE

We wish to compare t=3 drugs used in the treatment of erectile dysfunction with respect to length of effect. If we are given that the vector of treatment means equals

$$\bar{y} = [1.2, 1.3, 2.1] \quad (2)$$

and that the sample covariance matrix equals

$$\mathbf{S} = \begin{bmatrix} 2.3 & 2.2 & 1.4 \\ 2.2 & 2.5 & 1.9 \\ 1.4 & 1.9 & 2.4 \end{bmatrix}, \quad (3)$$

then from Figure 1, we see that a sample of size $n=25$ will have power $\geq 87\%$ at the $\alpha = 0.05$ level of significance to reject the null hypothesis (e.g., mean duration of effect are equal for each drug) given that it is false.

SAS® CODE

The SAS® code used to compute the data points plotted in Figure 1 is shown in Figure 2.

Figure 2

```
proc iml worksize=500;
  start main;
    t=3;
    c={1 0 -1, 1 -1 0};
    ybar={1.2, 1.3, 2.1}`;
    s={2.3 2.2 1.4, 2.2 2.5 1.9, 1.4 1.9 2.4};
    do n=5 to 50 by 0.01;
      do alpha=0.001, 0.01, 0.025, 0.05, 0.10, 0.20;
        power=probt(sqrt(n*(c*ybar`)*inv(c*s*c`)*
          (c*ybar`))-sqrt(((t-1)*(n-1))/(n-t+1))*
          finv(1-alpha,t-1,n-t+1)),n-1);
        print n alpha power;
      end;
    end;
  finish;
run main;
```

CONCLUSION

By taking advantage of the covariance structure among variables, the multivariate approach provides an efficient means for testing treatment differences in a repeated measurements design. In this paper, we have derived a method for computing the exact power for this procedure based upon the Hotelling's T^2 -statistic. The method is a direct extension from univariate space and provides a generalized framework for a broader class of multivariate power computation.

REFERENCES

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Figure 1: Exact Power Analysis for Multivariate Repeated Measurements Test

$\bar{y} = [1.2 \ 1.3 \ 2.1]$, $S = [2.3 \ 2.2 \ 1.4, 2.2 \ 2.5 \ 1.9, 1.4 \ 1.9 \ 2.4]$

